

Quantum Robots and Quantum Computers

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Validation of a presumably universal theory, such as quantum mechanics, requires a quantum mechanical description of systems that carry out theoretical calculations and systems that carry out experiments. The description of quantum computers is under active development. No description of systems to carry out experiments has been given. A small step in this direction is taken here by giving a description of quantum robots as mobile systems with on board quantum computers that interact with different environments. Some properties of these systems are discussed. A specific model based on the literature descriptions of quantum Turing machines is presented.

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I. INTRODUCTION

Much of the impetus to study quantum computation, either as networks of quantum gates [1,2] (See [3] for a review) or as Quantum Turing Machines [4–8], is based on the increased efficiency of quantum computers compared to classical computers for solving some important problems [9,10]. Realization of this goal or use of quantum computers to simulate other physical systems [11,6] requires the eventual physical construction of quantum computers. However, as emphasized repeatedly by Landauer [12], there are serious obstacles to such a physical realization.

There is, however, another reason to study quantum computers that is less dependent on whether or not such machines are ever built. It is based on the fact that testing the validity of a physical theory such as quantum mechanics requires the comparison of numerical values calculated from theory with experimental results. If quantum mechanics is universally valid (and there is no reason to assume otherwise), then both the systems that carry out theoretical calculations and the systems that carry out experiments must be described within quantum mechanics. It follows that the systems that test the validity of quantum mechanics must be described by the same theory whose validity they are testing. That is quantum mechanics must describe its own validation to the maximum extent possible [13].

Because of these self referential aspects, limitations in mathematical systems expressed by the Gödel theorems lead one to expect that there may be interesting questions of self consistency and limitations in such a description. Limitations on self observation by quantum automata [14–16] may also play a role here.

In order to investigate these questions it is necessary to have well defined completely quantum mechanical descriptions of systems that compute theoretical values and of systems that carry out experiments. So far there has been much work on quantum computers. These are systems that can, in principle at least, carry out computation of theoretical values for comparison with experiment. However there has been no comparable development of a quantum mechanical description of robots. These are systems that can, in principle at least, carry out experiments.

Another reason quantum robots are interesting is that it is possible that they might provide a *very small* first step towards a quantum mechanical description of systems that are aware of their environment, make decisions, are intelligent, and create theories such as quantum mechanics [17–19]. If quantum mechanics is universal, then these systems must also be described in quantum mechanics to the maximum extent possible.

The main point of this paper is that quantum robots and their interactions with environments may provide a well defined platform for investigation of many interesting questions generated by the above considerations. To this end some aspects of quantum robots and their interactions with environments are discussed in the next section. The close relation between quantum robots and quantum computers is clear from the definition of a quantum robot as a mobile system consisting of an on board quantum computer and needed ancillary systems that moves in and interacts with an environment.

A specific model of quantum robots plus environments is discussed in Section III. The model, which is based on the description of quantum Turing machines, describes the motion of a quantum robot in an environment which is a 1-D lattice of qubits. The overall model system consisting of a quantum robot plus environment is considered to be isolated with dynamics given by a time independent self adjoint Hamiltonian. The model is in essence also a slowed down version of a quantum Turing machine which is described so that it can be easily reinterpreted as a quantum

robot interacting with an environment. This interpretation is facilitated by separation of the step operator, defined in other work [6,8], into two parts describing action and computation phases.

In the last section some similarities and differences between quantum robots plus environments and quantum computers are discussed. Quantum robots which function as quantum computers by use of states of systems, that are a part of the environment, to represent numbers are seen to be limited in that there are environments in which a satisfactory number representation is not possible. Also the speculative possibility of a Church Turing type hypothesis for the class of physical experiments is noted.

It must be emphasized that the language used in this paper to describe quantum robots is carefully chosen to avoid any suggestions that these systems are aware of their environment, make decisions, carry out experiments or make measurements, or have other properties characteristic of intelligent or conscious systems. The quantum robots described here have no awareness of their environment and do not make decisions or measurements. Their description differs in detail only, from that used to describe any other system in quantum mechanics.

It should be noted that some aspects of the ideas presented here have already occurred in earlier work. Physical operations have been described as instructions for well-defined realizable and reproducible procedures [20], and quantum state preparation and observation procedures have been described as instruction booklets or programs for robots [21]. However these concepts were not described in detail and the possibility of describing these procedures or operations quantum mechanically was not mentioned. Also quantum computers had not yet been described. More recently Helon and Milburn [22] have described the use of the electronic states of ions in a linear ion trap as an apparatus (and a quantum computer register) to measure properties of vibrational states of the ions. In other work quantum mechanical Maxwell's demons have been described [23].

Also there is much work on the interactions between quantum computers and the environment. However, these interactions are considered as a source of noise or errors to be minimized or corrected by use of quantum error correction codes [24]. Here interactions between a quantum robot and the environment are emphasized as an essential part of the overall system dynamics. Other work on environmental induced superselection rules [25,26] also emphasizes interactions between the environment and a measurement apparatus that stabilize a selected basis (the pointer basis) of states of the apparatus.

II. QUANTUM ROBOTS

Here quantum robots are considered to be mobile systems that have a quantum computer on board and any other needed ancillary systems. Quantum robots move in and interact (locally) with environments of quantum systems. Since quantum robots are mobile, they are limited to be quantum systems with finite numbers of degrees of freedom.

Environments consist of arbitrary numbers and type of systems moving in 1-, 2-, or 3-dimensional spatial universes. The component systems can have spin or other internal quantum numbers and can interact with one another or be free. Environments can be open or closed. If they are open then there may be systems that remain for all time outside the domain of interaction with the quantum robot that can interact with and establish correlations with other environment systems in the domain on the robot. Quantum field theory may be useful to describe environments containing an infinite number of degrees of freedom. To keep things simple, in this paper environments will be considered to consist of systems in discrete space lattices instead of in continuous space.

The quantum computer that is on board the quantum robot can be described as a quantum Turing machine, a network of quantum gates, or any other suitable model. If it is a quantum Turing machine, it consists of a finite state head moving on a finite lattice of qubits. The lattice can have distinct ends. However it seems preferable if the lattice is closed (i.e. cyclic). If the on board computer is a network of quantum gates then it should be a cyclic network with many closed internal quantum wire loops and a limited number of open input and output quantum wires (narrow bandwidth). Even though acyclic networks are sufficient for the purposes of quantum computation [27] cyclic ones are preferable for quantum robots. One reason is that interactions between these networks and the environment are simpler to describe and understand than those containing a large number of input and output lines. Also the only known examples of *very* complex systems that are aware of their environment and are presumably intelligent, contain large numbers of internal loops and internal memory storage.

For the purposes of this paper the overall dynamics of a quantum robot and its interactions with the environment is described in terms of *tasks*. A task for a quantum robot is equivalent to a function associated with a quantum computer. A quantum robot carries out a task on some initial state of the environment just as a quantum computer carries out a function computation on a specified initial state.

The goal of the task is to change the initial environmental state into some final state with properties corresponding to the goal. An example of a task is "move each system in region R 3 sites to the right if and only if the destination site is unoccupied." Implementation of such a task requires specification of a path to be taken by the quantum robot

in executing the task. Some method of determining when it is inside or outside of the specified region and making appropriate movements must be available. In this case if there are n systems in region R at locations x_1, x_2, \dots, x_n in region R then the initial state of the regional environment, $|\underline{x}\rangle = \otimes_{j=1}^n |x_j\rangle$ becomes $\otimes_{j=1}^n |x_j + 3\rangle$ provided all destination sites are unoccupied.

If the initial state of the regional environment is a linear superposition of states $\psi = \sum_{\underline{x}} c_{\underline{x}} |\underline{x}\rangle$ of n -system position states $|\underline{x}\rangle$ in R then the final state of the regional environment is given in general by a density operator even if all destination sites are unoccupied. This is a consequence of the fact that in general the actions of the quantum robot introduce correlations between the states of the robot systems and the different initial environment component states $|\underline{x}\rangle$. When the task is completed on all components $|\underline{x}\rangle$, the overall state of the robot plus environment is given by a linear sum over robot regional environment states of the form $\sum_{\underline{x}} c_{\underline{x}} \theta_{\underline{x}} |\underline{x}\rangle$. Here $\theta_{\underline{x}}$ is the final state of the quantum robot resulting from carrying out the task on the regional environment in state $|\underline{x}\rangle$. Taking the trace over the robot system variables gives the density operator form for the regional environment state.

The above description shows that quantum robots can carry out the same task on many different environments simultaneously. This can be done by use of an initial state of the quantum robot plus environment that is a linear superposition of different environment basis states. For quantum computers the corresponding property of carrying out many computations in parallel has been known for some time [6]. Whether the speedup provided by this parallel tasking ability can be preserved for some tasks, as is the case for Shor's [9] or Grover's algorithms [10] for quantum computers, remains to be seen.

The above described task is an example of a reversible task. There are also many tasks that are irreversible. An example is the task "clean up the region R of the environment" where "clean up" has some specific description such as "move all systems in R to some fixed pattern". This task is irreversible because many initial states of systems in R are taken into the same final state. This task can be made reversible by storing somewhere in the environment outside of R a copy of each component in some basis B of the initial state of the systems in R . For example if $B = \{|\underline{x}\rangle\}$ and $\sum_{\underline{x}} c_{\underline{x}} |\underline{x}\rangle$ is the initial state, then the copy operation is given by $\sum_{\underline{x}} c_{\underline{x}} |\underline{x}\rangle |0\rangle_{cp} \longrightarrow \sum_{\underline{x}} c_{\underline{x}} |\underline{x}\rangle |\underline{x}\rangle_{cp}$.

This operation of copying relative to the states in some basis avoids the limitations imposed by the no-cloning theorem [32] because an unknown state is not being copied. The price paid is that copying relative to some basis introduces branching into the process in that correlations are introduced between the state of systems in the copy region and states of systems in R . This is the quantum mechanical equivalent of the classical case of making a calculation of a many-one function reversible by copying and storing the input [28].

In the above case carrying out the cleanup on the state $\sum_{\underline{x}} c_{\underline{x}} |\underline{x}\rangle |\underline{x}\rangle_{cp}$ corresponds to the operation $\sum_{\underline{x}} c_{\underline{x}} |\underline{x}\rangle |\underline{x}\rangle_{cp} \longrightarrow |\underline{y}\rangle \sum_{\underline{x}} c_{\underline{x}} |\underline{x}\rangle_{cp}$ where $|\underline{y}\rangle$ is the clean up state for the region R . The overall process is reversible as it can be described by the transformation $\sum_{\underline{x}} c_{\underline{x}} |\underline{x}\rangle |0\rangle_{cp} \longrightarrow |\underline{y}\rangle \sum_{\underline{x}} c_{\underline{x}} |\underline{x}\rangle_{cp}$. If the final state of the quantum robot depends on the initial state of the systems in region R , then correlations remain and the overall transformation corresponding to carrying out the cleanup task is given by $\sum_{\underline{x}} c_{\underline{x}} |\underline{x}\rangle |0\rangle_{cp} \theta_i \longrightarrow |\underline{y}\rangle \sum_{\underline{x}} c_{\underline{x}} |\underline{x}\rangle_{cp} \theta_{\underline{x}}$. Here θ_i and $\theta_{\underline{x}}$ are the initial and final states of the quantum robot.

Each task is considered here to consist of a sequence of computation and action phases. The purpose of each computation phase is to determine what action the quantum robot should take. The input to the computation carried out by the on board quantum computer includes the local state of the environment and any other pertinent information, such as the output of the previous computation phase. During a computation phase the robot does not move or change the state of the environment. It does change the state of an on board ancillary system, the output system (o) whose state determines the action taken following completion of the computation.

During each action phase the quantum robot makes local changes in the environment state or moves on the lattice. It can carry out either or both of these types of steps. Depending on the model used, each action phase can consist of one step or several steps. (The specific model described in the next section includes multistep actions.) Here one step consists of the robot moving to at most an adjacent lattice site, or changing the state of the environment in the neighborhood of the quantum robot, or both. During an action phase the state of the (o) system, which determines the action to be carried out, and the state of the on board quantum computer, is not changed. Also the quantum robot may or may not observe the local environment. Examples of actions that do not and do require local observations are "rotate the qubit (as a spin system) by an angle ϕ " and "rotate the qubit by an angle ϕ only if it is in state $|0\rangle$. If the qubit is in state $|1\rangle$ move to an adjacent site."

The description of tasks carried out by quantum robots requires the use of completion or halting flags to determine when individual action and computation phases are completed as well as when the overall task is completed. Such flags are necessary if the overall quantum robot plus environment dynamics is described by a Hamiltonian because the unitarity of e^{-iHt} requires that system motion occurs somewhere even after the task is completed.

Note that there are many examples of tasks that never halt. Nonhalting of tasks can arise from several sources. The task may consist of a nonterminating sequence of computation and action phases. Or either a computation of an

action phase may never halt. An example of an action that is multistep, does not halt, and requires local environment interactions at each step is "move along a string of 0s until a 1 is found" carried out on a lattice of 0s only in the direction of motion.

III. A SPECIFIC MODEL OF QUANTUM ROBOTS PLUS ENVIRONMENTS

Here a specific model of quantum robots plus environments is described that illustrates the above material. The close relationship between quantum robots plus environments and quantum computers is shown by the fact that the model also describes a slowed down version of a quantum Turing machine. In order to have a model described entirely within quantum mechanics, the overall system of quantum robot plus environment will be considered to be isolated with dynamics given by a self adjoint time independent Hamiltonian. This avoids the presence of external agents to turn on and off successive segments of a time dependent Hamiltonian. Also, the model will be described using information bearing degrees of freedom only. The relevance of this for the development of quantum computers has been noted by LandauerLand1.

The models are based on an expansion of quantum Turing machines [4–6,30,8] to describe models of quantum robots in a 1-D lattice qubit environment. The models also provide a natural decomposition of each phase into one or more single steps. The expansion is straightforward as the models already describe a multistate head moving on and interacting with a 1-D qubit lattice.

Models of quantum Turing machines consist of a 1-D finite or infinite lattice of qubits and a multistate head. A computation basis B_C for the overall system of head and lattice consists of the states $\{|l, j, \underline{s}\rangle\}$. Here $|l, j\rangle$ denotes the head internal state and lattice position and $|\underline{s}\rangle = \otimes_{j=-\infty}^{\infty} |\underline{s}_j\rangle$ denotes the state of the lattice qubit systems. For an infinite lattice B_C is uncountably infinite unless some tail condition is imposed on $|\underline{s}\rangle$. An example [8] is that $|\underline{s}_j\rangle \neq |0\rangle$ for at most a finite number of j . This tail condition applies to all states in B_C .

Each QTM is described by a step operator T acting on the Hilbert space spanned by the basis B_C . T is required to satisfy locality and homogeneity conditions. That is [6],

$$\langle l', j', \underline{s}' | T | l, j, \underline{s} \rangle = 0 \text{ if } \begin{cases} \underline{s}', \underline{s} \text{ differ at positions } \neq j \\ |j' - j| > 1 \end{cases} \quad (1)$$

This expresses the locality condition in that single step changes in the state of the lattice qubits are limited to the qubit at the position of the head and the head can move at most one site to the right or left. Also the matrix element is independent of the value of j and depends on the difference $j' - j$ only (homogeneity).

If T describes finite time interval steps as is done in some models [6,30] then T is also required to be unitary and iterations of T or T^\dagger describe model evolution. This requirement is dropped in other work [8] in which T is used to construct a Hamiltonian according to Feynman's prescription [31],

$$H = K(2 - T - T^\dagger) \quad (2)$$

where K is a constant [5,8]. As H is self adjoint and time independent, the finite time operator $e^{-iHt/\hbar}$ is unitary.

These models of quantum Turing machines can be changed into models of quantum robots interacting with environments by requiring the head h_1 to consist of an on board quantum Turing machine and three other ancillary systems, a memory system (m), an output system (o), and a control qubit (c). The on board quantum Turing machine consists of another head h_2 moving on a closed (e.g. circular) track of N qubits. Figure 1 shows the complete system. The qubit lattice \mathcal{L}_1 is the environment of the quantum robot and \mathcal{L}_2 is the on board N qubit lattice. The location of h_1 on \mathcal{L}_1 is marked by an arrow.

In this model changes of the head internal state, which occur in a single step in quantum Turing machines, Eq. 1, become multistep computations carried out by the on board quantum computer in each computation phase. A computation basis for the on board quantum Turing machine has states of the form $|p, k, \underline{t}\rangle$ that show h_2 in internal state $|p\rangle$ at site k on \mathcal{L}_2 and the \mathcal{L}_2 qubits in state $|\underline{t}\rangle = \otimes_{i=1}^N |\underline{t}_i\rangle$. The three added systems are used to regulate and determine the actions and computations of the quantum robot. The memory (m) and output (o) systems are each described by an L dimensional Hilbert space. The control system (c) is a qubit. A reference basis set B_{omc} for the three systems has the form $|l_1\rangle_m |l_2\rangle_o |i\rangle_c$ where $l_1, l_2 = 0, 1, \dots, L-1$ and $i = 0, 1$.

The function of the three added systems (o), (m), and (c) is based on the separation of the step operator T_{QR} into the sum of two operators:

$$T_{QR} = T_a + T_c. \quad (3)$$

where T_a and T_c describe respectively single steps of actions and computations of the quantum robot. The dynamics of the systems is given by Eq. 2 with T_{QR} replacing T .

The on board quantum Turing machine begins a T_c computation with (o), (m), and (c) in state $|l_2\rangle_o|l_1\rangle_m|0\rangle_c$ where $|l_2\rangle$ and $|l_1\rangle_m$ are the respective output from and input to the previous computation. This state and the reference basis state $|s\rangle$ (with $s = 0, 1$) of the \mathcal{L}_1 qubit at the quantum robot location are the inputs to the computation.

The goal of each computation phase is the computation of a new state $|l_3\rangle_o$ of the output system and a shift of the input state of (o) to the memory system (m). The overall change of (o), (m), and (c) can be represented by $|l_2\rangle_o|l_1\rangle_m|0\rangle_c \longrightarrow |l_3\rangle_o|l_2\rangle_m|1\rangle_c$ which represents a change of the states of (o), (m), and (c) systems in a reference basis B_{omc} . The last step of the computation is the conversion of the control qubit state from $|0\rangle$ to $|1\rangle$ as T_c is active only if the control qubit is in state $|0\rangle$.

This description applies to those computation phase operators such that both T_c and T_c^\dagger take states of B_{omc} into states of B_{omc} . If either T_c or T_c^\dagger are such that iteration of these operators takes states of B_{omc} into linear superpositions of states in B_{omc} , then branchings or entanglements are introduced.

When the computation is finished the robot carries out the action described by T_a . The input to T_a is the state $|l_3\rangle_o|l_2\rangle_m|1\rangle_c$. The state $|l_3\rangle_o|l_2\rangle_m$ determines which action the robot will carry out and the state $|1\rangle_c$ activates the action phase of the robot. If the quantum robot completes the action, then the last step of T_a is to change the control qubit state from $|1\rangle$ back to $|0\rangle$. The (m) and (o) states are unchanged throughout the action provided the states belong to a reference basis such as B_{omc} . This restriction and that given above for T_c for the \mathcal{L}_1 qubit state avoid the limitations of the no cloning theorem [32]. This completes the cycle as T_c becomes active again.

The above description shows that the time evolution of the quantum robot proceeds by alternating computing and action phases each containing ≥ 1 step or iteration of T_a or T_c . One way to ensure that this proceeds smoothly is to require that, except for the memory, output, and control systems, the terminal state of the quantum Turing machine for one computation phase be the same as the initial state for the next computation phase. For example let T_c begin and end a computation with $|p, k, \underline{t}\rangle = |0, 0, \underline{0}\rangle$. The main function of the \mathcal{L}_2 qubit lattice is as a scratch pad for any calculation. If part of it is set aside for added memory or for input information, the above conditions on the initial and final states would be changed to accomodate this.

The types and properties of possible actions that the quantum robot can carry out in an action phase depend on the model being considered. They can be either single step (h_1 motion at most one \mathcal{L}_1 site) or multistep (h_1 motion of several sites). They also may or may not be mediated by observations of the environment. For instance the action "rotate the qubit by the angle ϕ " is a single step action that requires no observation. It applies to the qubit at the quantum robot location whatever its state is. The multistep action "move along a chain of 0s, changing each qubit state $|0\rangle$ to $a|0\rangle + b|1\rangle$ until a 1 is encountered" requires observation of the environmental qubit at each successive robot location to see if it is a 1 or a 0.

The descriptions and requirements given above can be given in terms of conditions that both T_a and T_c should satisfy. T_c is related to an operator \tilde{T}_c defined on the Hilbert space spanned by the basis set $\{|p, k, t, s, l_1, l_2, i\rangle\}$. Here $|t\rangle$ and $|s\rangle$ denote the states of the qubits at the locations of h_2 on \mathcal{L}_2 and h_1 on \mathcal{L}_1 respectively, and $|i\rangle$ denotes the state of (c). Let the states $|\theta'\rangle$ and $|\theta\rangle$ denote respectively the states $|p', k', s', l'_1, l'_2, i'\rangle$ and $|p, k, s, l_1, l_2, i\rangle$. One has

$$\langle\theta', \underline{t}'|T_c|\theta, \underline{t}\rangle = \langle\underline{t}'_{\neq k}|\underline{t}_{\neq k}\rangle\langle\theta', \underline{t}'_k|\tilde{T}_c|\theta, \underline{t}_k\rangle \quad (4)$$

Here $\langle\underline{t}'_{\neq k}|\underline{t}_{\neq k}\rangle$ denotes the product for all qubits in \mathcal{L}_2 not at position k . This condition states that \mathcal{L}_2 qubit changes are limited to the qubit at the location of h_2 .

The operator \tilde{T}_c satisfies the conditions

$$\tilde{T}_c = \sum_{k, k'} \sum_s P_{k'} P_s \tilde{T}_c P_s P_k P_0^c \quad (5)$$

where $P_1^c = |1\rangle_c\langle 1|$ and $P_k = |k\rangle\langle k|$. The prime on the k, k' -sum means that it is limited to values for which $|k' - k| = 0, 1$. Also the values of the matrix elements of \tilde{T}_c depend on the difference $k' - k$ and not on the value of k .

The equation states that when T_c is active the state $|s\rangle$ of the qubit at the location of h_1 is not changed. This is expressed by the requirement that T_c is diagonal in the projection operator $P_s = |s\rangle\langle s|$. Also single step motions of h_2 are limited to at most one site on \mathcal{L}_2 and \tilde{T}_c is active (nonzero) only when the control qubit is in state $|0\rangle$.

The operator T_a describes actions of the quantum robot h_1 on \mathcal{L}_1 . It is active in the Hilbert space spanned by the basis $\{|l_2, l_1, i, j, \underline{s}\rangle\}$ where $|l_2\rangle$, $|l_1\rangle$, and $|i\rangle$ are the respective states of the (o), (m), and (c) systems, and j is the position of h_1 on \mathcal{L}_1 . T_a satisfies,

$$\begin{aligned} \langle\phi', j', \underline{s}'|T_a|\phi, j, \underline{s}\rangle &= \langle\underline{s}'_{\neq j}|\underline{s}_{\neq j}\rangle\delta_{\underline{s}'_j, s'}\delta_{\underline{s}_j, s} \\ &\times \langle\phi', j', s'|T_a|\phi, j, s\rangle \end{aligned} \quad (6)$$

where $|phi\rangle = |l_2, l_1, i\rangle$. This condition states that changes in \mathcal{L}_1 qubits are limited to the qubit at the location of the quantum robot.

The operator \tilde{T}_a satisfies the conditions

$$\tilde{T}_a = \sum_{j', j} \sum_{l_1, l_2} P_{j'} P_{l_1}^m P_{l_2}^o \tilde{T}_a P_{l_2}^o P_{l_1}^m P_j P_1^c \quad (7)$$

The equation states that \tilde{T}_a is diagonal in the B_{omc} basis for the (o) and (m) systems only and thus does not change the (o) and (m) states provided they are in the B_{omc} basis. As noted this avoids the limitations of the no cloning theorem [32]. Also T_a is active only when $|j' - j| \leq 1$ and the control system is in state $|1\rangle$.

To avoid complications, the need for history recording has not been discussed. Both the computation and action phases may need to record some history. For example when T_c is active, the change $|l_2\rangle_o |l_1\rangle_m \longrightarrow |l_3\rangle_o |l_2\rangle_m$ requires history recording if the change is not reversible. Where records are stored (on h_1 or in the environment) depends on the model. Also the task carried out by the quantum robot may not be reversible unless the initial environment is copied or recovered.

Initial and final states for the starting and completion of tasks need to be described. For example at the outset the memory, output, and control systems might be in the state $||0\rangle_m |l_i\rangle_o |0\rangle_c |0\rangle_c$ and the environment would be in some suitable initial state. The process begins with the on board quantum computer active.

Completion of a task could be described by designating one or more states $|l_f\rangle$ as final output states and arranging matters so that the action of T_a based on any of these states moves the quantum robot along \mathcal{L}_1 with no changes in the environment state. As is the case for computation, continued motion of some type is necessary for any reversible process.

As noted the specific model of a quantum robot plus environment is in essence a slowed down version of a quantum Turing machine. Each step of a quantum Turing machine is replaced by a multistep quantum computation followed by a single action step. This replacement raises the question whether the model quantum robots plus environments are as powerful when used as quantum computers as the original quantum Turing machines.

This question is open. However one can show that if the dependence of T_a on the memory states is removed, then the specific models of quantum robots plus environments with dynamics given by Eq. 3 are weaker as quantum computers than quantum Turing machines described by Eq. 1. To see this let T be such that $T|l, j, \underline{s}\rangle = |l_1, j_1, \underline{s}_1\rangle$ and $T|l', j, \underline{s}\rangle = |l_1, j_2, \underline{s}_2\rangle$ where $l \neq l'$, $j_1 \neq j_2$, and $\underline{s}_1 \neq \underline{s}_2$. Modelling this with a quantum robot requires that T_a depend on the memory state. This is the reason that both the (o) and (m) systems are present in the model. However this dependence can be excluded if desired as there is no a priori reason why quantum robots plus environments, when functioning as quantum computers, should be as powerful as quantum Turing machines.

IV. DISCUSSION

The model described in the last section raises the question "Are there any real differences between quantum robots interacting with environments and quantum computers?" Here it will be seen that the answer is that there are real differences. To begin with one notes that quantum robots plus environments can function as quantum computers in two ways. One obvious way is by use of the quantum computer on board the robot as a stand alone quantum computer with no environment-robot interactions needed. The other way uses states of a collection C of systems in the environment to construct a k-ary representation of numbers with $k \geq 2$. In this case the quantum robot and the systems in C are the quantum computer with interactions between the quantum robot and the systems in C generating the steps in the quantum computation. The fact that the quantum robot includes an on board quantum computer is not relevant provided one is only interested in using the system (quantum robot plus C) as a quantum computer. In the models described in section III the fact that states of the environment as a lattice of qubits can be used to represent numbers is incidental to consideration of the system as a quantum robot plus environment.¹

This equivalence between quantum robots plus environments and quantum computers holds only for those environments containing collections C of systems as described above. However there are many environments that do not

¹It may be possible to carry these ideas over to quantum computers represented by networks of quantum gates. In this case the network may be reinterpreted as a community of motionless, very simple quantum robots interacting with an environment of moving qubits. Here also the collection C is the whole environment. Such an interpretation depends on considering a single quantum gate as a very simple quantum computer.

contain any such collection C of systems. Examples include environments of interacting moving quantum systems or environments of systems whose only possible number representation is unary. An example of the latter is a single spinless system on a 1-D space lattice. Here the number representation is given by the distance or number of sites from an origin to the system on the lattice. The head interacting with this environment does not correspond to an efficient quantum computer. On the other hand this example, and the example of moving interacting systems are acceptable environments for a quantum robot. Note that no requirement of efficiency is imposed for quantum robots.

The above illustrates one of the differences between quantum robots interacting with environments and quantum computers. Another difference is a matter of emphasis. For quantum robots the emphasis is on the quantum robot interacting with and changing the state of external systems that are not part of the robot. Because the systems are external to the robot and are part of the environment, problems with self observation by quantum automata [14–16] do not arise.

For quantum computers the interactions between the systems whose states are used to represent numbers and other computer components are internal to and an essential part of the computer. Effects of external systems are to be minimized or corrected for [24]. This is the case whether these systems are part of the quantum robot (as the head) or are part of the environment. Here problems with self observation may arise because systems that are external to the system as a quantum robot plus environment are now internal to and part of the quantum computer.

In conclusion the following speculative ideas may be worth considering. The close connection between quantum computers and quantum robots interacting with environments suggests that the class of all possible physical experiments may be amenable to characterization just as is done for the computable functions by the Church-Turing hypothesis [33,6,34]. That is there may be a similar hypothesis for the class of physical experiments.

The description of tasks carried out by quantum robots (Section II) lends support to this idea in that there may be an equivalent Church Turing hypothesis for the collection of all tasks that can be carried out. The earlier work that characterizes physical procedures as collections of instructions [20,35], or state preparation and observation procedures as instruction booklets or programs for robots [21] also supports this idea. On the other hand much work needs to be done to give a precise characterization of physical experiments, if such is indeed possible.

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FIGURE CAPTION

Figure 1. A Model of a Quantum Robot and its Environment. The environment is an infinite 1-D lattice \mathcal{L}_1 of qubits. The quantum robot h_1 consists of an on board Quantum Turing machine, finite state memory (m) and output (o) systems, and a control qubit (c). The on board QTM consists of a finite closed lattice \mathcal{L}_2 of qubits and a finite state head h_2 that moves on \mathcal{L}_2 . The position of the quantum robot h_1 on the environment lattice \mathcal{L}_1 is shown by an arrow.

